

How to implant a causal Θ function into a tachyon field operator

V.F. Perepelitsa

ITEP, Moscow

Abstract

The preferred reference frame in which the signal propagation is governed by retarded causality is a must for any theory of faster-than-light particles and signals. It is shown in previous papers of the author that such a system does exist and is the comoving system of the general relativity branch, the relativistic cosmology. Restrictions imposed by the causality conservation can be expressed via a causal Θ function assumed to be acting in both, the classical and quantum field theories of tachyons. A Lorentz-covariant introduction of this Θ function, which ensures a causal behaviour of real tachyons (asymptotic tachyonic states) preventing the appearance of casual loops constructed with the use of faster-than-light particles and signals, into the tachyon quantum field operators is suggested in this note.

1 Introduction

During the late fifties and sixties of the last century an idea of a possibility of the introduction of a concept of faster-than-light particles into the quantum field theory was considered in several papers [1, 2, 3, 4, 5, 6]. The particles were called tachyons, from the Greek word $\tau\alpha\chi\iota\sigma$ meaning *swift* [5]. These considerations have generated a strong critical response based on the generally accepted principles of causality [7, 8, 9], vacuum stability [10] and unitarity [11, 12]. A consensus was achieved that within the special relativity and the canonical quantization procedure faster-than-light particles are incompatible with those principles.

In ref. [13] a Lorentz-covariant approach based on the requirement of the causality non-violation was suggested, which solved also two other problems of tachyon models, i.e. the problem of the tachyon vacuum instability and the problem of a violation of unitarity by interacting tachyons. The causality violation by tachyons was removed by the introduction of a preferred reference frame in which the events of tachyon exchange are ordered by retarded causality, which ensures the absence of causal loops in any frame. It was shown in [13] that the preferred reference frame considered there should be associated with the so called comoving frame (see the definition of the latter, for example, in [14]), being a universal reference frame in which our universe is embedded. In particular, the distribution of matter in the universe is isotropic in this frame only, the same is true for the relic black body radiation. The introduction of the preferred frame recovers, as a straightforward consequence, the concept of absolute time.

The causality protection formula, valid in all inertial frames, was formulated in [13] as follows:

$$Pu \geq 0, \tag{1.1}$$

where P is a 4-vector of particles transferring a signal and u is a 4-velocity of the preferred reference frame with respect to (any particular) inertial observer. It is a boundary condition which should be imposed on solutions of any tachyon equation of motion.

As a straightforward result, it turns out that the negative energy tachyons, which could be used for a construction of the causal loops, cannot appear in the preferred reference frame since the eigenvalues of the tachyon Hamiltonian are restricted from below, in this frame, by zero value, which automatically solves the problem of the tachyon vacuum instability. The tachyon vacuum in the preferred frame is represented by an ensemble of zero-energy, but finite-momentum, on-mass-shell tachyons propagating isotropically. Thus the space of the preferred frame is spanned by the continuous background of free, zero-energy tachyons; in some respects this is the reincarnation of the ether concept in its tachyonic version. Simultaneously it turns out that in any reaction in which tachyons participate asymptotic “in” and “out” tachyonic Fock spaces are unitarily equivalent, which removes the unitarity problem.

As “toy” models, the Lorentz-covariant quantum field models of scalar tachyons were considered in ref. [13]. They are based on Lorentz-invariant Lagrangians with spontaneously broken Lorentz symmetry, so the Lorentz invariance violation appears to be restricted to the tachyon sector only, affecting the asymptotic tachyon states and leaving the sector of ordinary particles within the Standard Model untouched, at least up to possible very small radiative corrections. The basic element of those tachyon models is the Lorentz-covariant causal Θ -function, required by the boundary condition (1.1), which ensures the causal behaviour of tachyonic fields and, subsequently, the other gains of the

models. For example, the Hermitian tachyon field operator with this Θ -function, $\Theta(ku)$, reads as follows:

$$\Phi(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^4k \left[a(k) \exp(-ikx) + a^\dagger(k) \exp(ikx) \right] \delta(k^2 + \mu^2) \Theta(ku), \quad (1.2)$$

where k is a tachyon four-momentum, $a(k), a^\dagger(k)$ are annihilation and creation operators with bosonic commutation rules, annihilating or creating tachyonic states with 4-momentum k , and μ is a tachyon mass parameter. As can be seen, the expression (1.2) is explicitly Lorentz-covariant. This covariance includes the invariant meaning of the creation and annihilation operators defined in the preferred frame; thus, for example, an annihilation operator $a(k)$ remains an annihilation operator $a(k')$ in the boosted frame, even if the zero component of k' may become negative. This is because the one-sheeted tachyon mass-shell hyperboloid is divided by the covariant boundary $\Theta(ku)$ into two parts separated in an invariant way. This results, in particular, in a possibility of the standard operator definition of the invariant vacuum state $|0\rangle$ via the annihilation operators $a(k)$, $a(k)|0\rangle = 0$ for all k , because the field Hamiltonian turns out to be bounded from below in any reference frame. For example, in the frames moving along the tachyon velocity these boundaries (i.e. the boundaries of the tachyon vacuum) are given by expressions

$$E_0^+ = \frac{\mu|\mathbf{u}|}{\sqrt{1-\mathbf{u}^2}} \quad (1.3)$$

for the direction coinciding with the tachyon velocity and by

$$E_0^- = -\frac{\mu|\mathbf{u}|}{\sqrt{1-\mathbf{u}^2}}. \quad (1.4)$$

in the frames moving in the opposite direction.

When calculating the tachyon production probabilities and cross-sections the confining Θ functions will accompany the production amplitudes as factors restricting the reaction phase space, so the expressions for these probabilities can be displayed as follows:

$$W = \int |M|^2 d\tau \prod_i \Theta(k_i u), \quad (1.5)$$

where M is a matrix element of the reaction (which has to be representable in a Lorentz-invariant form), $d\tau$ is a reaction phase space element, and the product of Θ functions includes all free tachyons (having 4-momenta k_i) participating in the reaction.

During discussions of the models above the author was asked to formalize the introduction of the causal Θ function into the tachyon field operator [15]. This note is assigned to realize this recommendation.

In formulae used in the note the velocity of light c and the Planck constant \hbar are taken to be equal to 1.

2 Causal tachyon field operator

Let us consider a Lorentz-invariant Lagrangian of a free scalar tachyon field

$$L = \frac{1}{2} \int d^3\mathbf{x} \left[\dot{\Phi}^2(x) - \nabla\Phi(x)\nabla\Phi(x) + \mu^2\Phi^2(x) \right] \quad (2.1)$$

from which the Klein-Gordon equation with the negative mass-squared term $-m^2 = \mu^2$ follows:

$$\left(\frac{\partial^2}{\partial t^2} - \partial_i \partial^i - \mu^2\right) \phi(x) = 0, \quad i = 1, 2, 3 \quad (2.2)$$

It has well-known solutions in the form of plane waves, so the wave function of a free particle with a given 4-momentum $k = (\omega, \mathbf{k})$ must be

$$const \times \exp -i(\omega t - \mathbf{k}\mathbf{x}) \quad (2.3)$$

with the dispersion relation

$$\omega = \pm \sqrt{\mathbf{k}^2 - \mu^2} \quad (2.4)$$

and with the restriction on the particle 3-momentum \mathbf{k} [3, 5]:

$$|\mathbf{k}| \geq \mu. \quad (2.5)$$

A Fourier representation of the general solution $\phi(x)$, up to a normalization factor $1/\sqrt{(2\pi)^3}$, should be written as

$$\phi(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi(k), \quad (2.6)$$

where $\delta(k^2 + \mu^2)$ ensures that the field $\phi(x)$ corresponds to free particles positioned on the mass shell, thus validating (2.5), and $\phi(k)$ are Fourier amplitudes. A (standard) problem appearing at such a decomposition is related to the negative sign of the ω in (2.4) which, being interpreted in a straightforward way as a particle energy, would lead to a well-known problem related to particles with negative energies.

Our aim is a standard solution of this problem combined with a “soft” (covariant) introduction of a concept of the preferred reference frame in the tachyon field model. To do this we introduce two auxiliary scalar fields that obey the equation (2.2):

$$\phi^{(+)}(x) = \frac{1}{2\pi i} \int_{C_+} \phi(x - u\tau) \frac{d\tau}{\tau}, \quad (2.7)$$

$$\phi^{(-)}(x) = \frac{1}{2\pi i} \int_{C_+} \phi(x + u\tau) \frac{d\tau}{\tau}, \quad (2.8)$$

where τ is a “time” parameter (which will be explained below), u , primarily, is some 4-vector of dimension of a 4-velocity, and the contour C_+ is extended from $-\infty$ to $+\infty$, deformed below the singularity at $\tau = 0$. These auxiliary fields are similar to those introduced in [16] in order to define invariantly the plane waves with positive and negative “frequencies”. In [16] a 4-vector ϵ , formally defined to be timelike and to have a positive time component, was used instead of the u . In our consideration the 4-vector u has a physical meaning of the 4-velocity of the preferred reference frame with respect to the observer, i.e. it is automatically timelike and has a positive time component:

$$u_\mu u^\mu = 1, \quad u_0 = u^0 > 0, \quad \mu = 0, 1, 2, 3. \quad (2.9)$$

The physical meaning of integrals in (2.7), (2.8) implies the “collection” of all virtually allowed field phases (“trajectories”) of each individual field mode, dispersed over absolute time τ . According to the decomposition (2.6),

$$\phi^{(+)}(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(+)}(k) \frac{1}{2\pi i} \int_{C_+} \exp(i k u \tau) \frac{d\tau}{\tau}, \quad (2.10)$$

$$\phi^{(-)}(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \frac{1}{2\pi i} \int_{C^+} \exp(-i ku \tau) \frac{d\tau}{\tau}. \quad (2.11)$$

Noting that

$$\frac{1}{2\pi i} \int_{C^+} \exp(i ku \tau) \frac{d\tau}{\tau} = \begin{cases} 1 & \text{if } ku > 0 \\ 0 & \text{if } ku < 0 \end{cases} \quad (2.12)$$

and

$$\frac{1}{2\pi i} \int_{C^+} \exp(-i ku \tau) \frac{d\tau}{\tau} = \begin{cases} 1 & \text{if } ku < 0 \\ 0 & \text{if } ku > 0 \end{cases} \quad (2.13)$$

the equations (2.10), (2.11) can be rewritten as

$$\phi^{(+)}(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(+)}(k) \Theta(ku), \quad (2.14)$$

$$\phi^{(-)}(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \Theta(-ku). \quad (2.15)$$

Let us consider these equations in the preferred reference frame, where $u = (1, 0, 0, 0)$:

$$\phi^{(+)}(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(+)}(k) \Theta(\omega), \quad (2.16)$$

$$\phi^{(-)}(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \Theta(-\omega). \quad (2.17)$$

One can see that in the preferred frame $\phi^{(+)}(x)$ differs from zero only at positive “frequencies” (the upper sign for ω in (2.4)), while $\phi^{(-)}(x)$ differs from zero only at negative “frequencies” (the lower sign for ω in (2.4)). Introducing the definition $E = \omega$ in (2.16) we can return to the covariant expression (2.14) for $\phi^{(+)}(x)$, where k is redefined as

$$k = (E, \mathbf{k}). \quad (2.18)$$

Analogously, introducing the definition $E = -\omega > 0$ in (2.17) and using the identity

$$\int d^3\mathbf{k} \exp(i\mathbf{k}\mathbf{x}) \int_{C^+} \exp[i(Eu_o + \mathbf{k}\mathbf{u})\tau] \frac{d\tau}{\tau} = \int d^3\mathbf{k} \exp(-i\mathbf{k}\mathbf{x}) \int_{C^+} \exp[i(Eu_o - \mathbf{k}\mathbf{u})\tau] \frac{d\tau}{\tau} \quad (2.19)$$

we can rewrite (2.15) as

$$\phi^{(-)}(x) = \int d^4k \exp(ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \Theta(ku) \quad (2.20)$$

with the definition (2.18) for k .

Thus we have arrived at a result that in both functions, $\phi^{(+)}(x)$ and $\phi^{(-)}(x)$, the variable E , which will be referred to in what follows as a particle energy, is, by the construction, always positive in the preferred reference frame. Keeping this in mind, we can write a general solution of (2.2) in an arbitrary frame in the form

$$\phi(x) = \int d^4k [\exp(-ikx) a(k) + \exp(ikx) a^+(k)] \delta(k^2 + \mu^2) \Theta(ku), \quad (2.21)$$

the amplitudes $\phi^{(+)}(k)$, $\phi^{(-)}(k)$ in the previous expressions for $\phi^{(+)}(x)$, $\phi^{(-)}(x)$ being replaced, by a procedure of second quantization, by annihilation and creation operators $a(k)$, $a^+(k)$, respectively, annihilating and creating states with 4-momentum k . The meanings of the operators $a(k)$, $a^+(k)$ to be the annihilation and creation operators are conserved in an arbitrary frame, even if the energy component of a 4-vector k (i.e. E) can become negative in a boosted frame under a suitable proper Lorentz transformation.

3 Chronology protection agency at work

It is interesting to note that formula (1.1) works in the case of ordinary particles also, destroying the possibility of having negative energies of these particles. This suggests the idea that this formula has a general application and can be considered as a realization of “the chronology protection agency”, the term being primarily introduced by S. Hawking [17] to protect the causality in some general relativity applications. The efforts undertaken by great physicists in the first middle of 20th century to overcome the problem of negative particle energies appearing in relativistic theory may be considered as being equivalent to the introduction of the concept of the preferred reference frame (and hence “absolute time”) in the philosophy of that theory, which trivially solves the problem, analogously to the prescriptions (2.7), (2.8) for a scalar tachyon field.

To make this story complete let us remark that the need of an introduction of the concept of the preferred reference frame in the quantum field theory in order to ensure the conservation of causality was noticed long time ago by P. H. Eberhard in [18].

4 Conclusion

The introduction of the causal Θ function $\Theta(ku)$ into the tachyon field operator, where k is a tachyon 4-momentum and u is the velocity of the preferred reference frame in which tachyon interactions are ordered by retarded causality, aimed at solving several serious problems of a theory of faster-than-light particles, is formalized in this note within a Lorentz-covariant approach.

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